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Estimating Candidate Support in Voting Rights Act Cases: Comparing Iterative EI and EI-R×C Methods

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Abstract

Scholars and legal practitioners of voting rights are concerned with estimating individual-level voting behavior from aggregate-level data. The most commonly used technique, King's ecological inference (EI), has been questioned for inflexibility in multiethnic settings or with multiple candidates. One method for estimating vote support for multiple candidates in the same election is called ecological inference: row by columns (R×C). While some simulations suggest that R×C may produce more precise estimates than the iterative EI technique, there has not been a comprehensive side-by-side comparison of the two methods using real election data that analysts and legal practitioners often rely upon in courts. We fill this void by comparing iterative EI and R×C models with a new statistical package—eiCompare—in a variety of R×C combinations including 2 candidates and 2 groups, 3 candidates and 3 groups, and up to 12 candidates and three groups and multiple

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candidates and four groups. Additionally, we examine the two methods with 500 simulated data sets that differ in combinations of heterogeneity, polarization, and correlation. Finally, we introduce a new model congruence score to aid scholars and voting rights analysts in the substantive interpretation of the estimates. Across all of our analyses, we find that both methods produce substantively similar results. This suggests that iterative EI and R×C can be used interchangeably when assessing precinct-level voting patterns in Voting Rights Act cases and that neither method produces bias in favor or against finding racially polarized voting patterns.

Keywords

ecological inference, statistical methods, aggregate data, voting, elections

American politics scholars and the U.S. court system commonly assess whether racially polarized voting (RPV) exists in a particular jurisdiction—whether a legislative district, city district, or county supervisor seat—as part of a voting rights analysis. Key's (1949) seminal study of Southern politics documented that Anglos (whites) living around high percentages of blacks voted most consistently for racially hostile Anglo candidates. Since then, extensive research has demonstrated that African Americans, Latinos, and Anglos disproportionately favor co-ethnic candidates and exhibit different preferences and voting patterns (Barreto 2007, 2010; Barreto, Villarreal, and Woods 2005; Dawson 2003; Grofman 1991; Grofman and Handley 1989; Grofman and Migalski 1988; Issacharoff 1992; McCrary 1990; Piston 2010; Tate 1994). With the passage of the 1965 Voting Rights Act (VRA) and subsequent amendments and court decisions, systematic examination of RPV patterns not only became increasingly relevant to scholars of race and ethnic politics but also the courts and legal practitioners as one major goal of the law was to increase African American voter registration and representation (Cox and Miles 2008; Davidson 1994; Lublin 2004). While the VRA contributed to increasing black voter registration (David- son 1994), and eventually descriptive representation (Grose 2011; Guinier 1991; Lublin 1999), gerrymandering and RPV in some localities still prevent minorities from electing their preferred candidates into office. As such, the courts are still concerned with determining whether various jurisdictions violate portions of the VRA.

In *Thornburg v. Gingles, 478 U.S. 30, 1986*, the court established a legal framework to guide VRA challenges to legislative districts or at-large voting systems that have been accused of diluting minority voting opportunities. According to *Gingles*, there are three prongs that plaintiffs must establish through an analysis of voting data to make a successful claim: (1) the minority group is both geographically compact and large enough to create a single-member district, (2) the minority group tends to vote together and is politically cohesive, and (3) the nonminority (majority group) tends to vote in the opposite direction, such that it can usually block the minority groups' preferred candidate (Ross 1993). Based on this framework and the court's prescribed statistical methods (Grofman 1992), social scientists were asked to employ voting analyses by relying on a combination of precinct voting data and voter demographic data, often derived from Census, surname matching, or bayesian improved surname geocoding (BISG) data (Imai and Khanna 2016) to assess whether a jurisdiction is in violation of the VRA.[1] At the most basic level, an analysis of ecological voting data aided the courts in answering the following important question: Do Anglos block vote against African American candidates and prevent African Americans from gaining political representation?

Using more simple methods, the early evidence presented at trial supported what Key had already found (e.g., Goodman 1953, 1959). Over the decades, racial demographics and social science tools have evolved considerably. King (1997) and Grofman (1992, 1995), for instance, advocated for a more precise measurement of racial voting patterns beyond homogeneous precinct analysis, simple correlation techniques, and Goodman's regression. No longer facing a strictly Black-Anglo hypersegregated environment, others, notably Rosen et al. (2001), advocated for methods to account for an increase in racially heterogeneous neighborhoods and the rapid emergence of Latinos and Asians.

As it stands, social scientists—and the courts—most often rely on two specific statistical approaches to ecological data.[2] The first, iterative ecological inference (EI), developed by King (1997), was originally created for analysis of two racial or ethnic groups, and potentially only two candidates contesting one seat. The second and computationally intensive approach, EI row by column (R×C), developed by Rosen et al. (2001), was developed for instances when there are multiple racial or ethnic groups, or multiple candidates contesting office. While these methods make unique contributions, it is, however, unclear whether both would produce *substantively* different results when faced with the exact same real-world voting data set. In one case, Grofman and Barreto (2009) used multiple ecological approaches on the same data set and did not find any substantive or statistical differences (for similar comparisons, also see de Benedictis-Kessner 2015). However, others have argued that using King's iterative EI technique with multiple racial groups or multiple candidates may produce biased estimates (Ferree 2004). Other social scientists have gone even further, asserting in court that the iterative EI approach cannot be used to analyze multiple racial group or multiple candidate elections because "... it biases the analysis for finding racially polarized voting" (Katz 2014, p. 13).

As with any methodological advancement, there is often a debate in the literature. However, very little real election data have been brought to bear. Ferree (2004) assessed King's iterative approach with simulated data and a parliamentary election in South Africa using a proportional representation system. Grofman and Barreto (2009) compared an exit poll to precinct election data in Los Angeles (LA), but only compared Goodman's ecological regression against King's iterative EI without evaluating the R×C approach. We contribute to this literature with a comprehensive analysis of real ecological voting data from 14 elections and 78 candidates in multiethnic settings across the United States.

Using real-world ecological voting data, we aim to answer three fundamental questions not previously addressed: (1) Does the iterative EI method substantially overestimate RPV compared to R×C? In other words, does iterative EI bias the results toward detecting RPV? (2) Are there systematic differences in the outcomes produced by iterative EI and R×C when analyzing elections with few candidates versus elections with multiple candidates? and (3) Are there systematic differences when analyzing elections with more than two racial groups?

With regard to the last two questions, if R×C is indeed a "better" method for assessing group voting behavior in a multicandidate context, then one should expect to see substantively different estimates across the two methods. Specifically, relative to R×C, the iterative EI method should become unstable and possibly generate completely different estimates in scenarios with multiple candidates and/or multiple racial/ethnic groups. Our analysis does not find this to be the case. Instead, we find strong patterns of consistency across iterative EI and R×C despite claims to the contrary. Across the 78 candidates and 193 vote choice scenarios we analyzed, there is no convincing evidence that either iterative EI or R×C is biased toward or against findings of RPV. Further, the point estimates that both methods produce are remarkably similar, typically within two points of one another. For social scientists and legal scholars interested in analyzing RPV when only ecological data are present, both approaches can be relied upon as they lead to substantively similar conclusions about the presence or absence of RPV. Additional systematic analysis with simulated data sets provides additional evidence in support of our assertions. While our examination is fairly comprehensive and in line with other published works that compare different methods (Brown and Dunn 2011; Burnham and Anderson 2004; Markovsky and Eriksson 2012; Muthén and Asparouhov 2017), we encourage future research to extend the bounds of our study to further examine similarities or differences between iterative EI and R×C as it pertains to RPV analysis.

In the sections that follow, we first review the literature on EI that is relevant to RPV analysis. Second, we describe the data sets gathered across several states spanning more than a decade. The first few data sets all contain elections in areas with relatively high Latino (and Anglo) voting populations and contain at least one Spanish-surnamed candidate. In addition to Latinos, many of the data sets include sizable African American and Asian American populations, which allows us to examine how iterative EI and R×C operate in different racial and ethnic contexts. We also examine elections with 2, 3, 4, and up to 12 different candidates to fully assess how both models work in different electoral environments. Beyond this, we demonstrate that both the iterative EI and the R×C methods produce results in line with individual-level exit poll data. We then present Monte Carlo simulation results and introduce a congruence analysis based on a simple 2 × 2 comparison that can be applied to multiple groups and candidates to highlight the ways in which analysts can determine whether the two aforementioned methods result in the same substantive conclusion. Finally, we conclude with a brief discussion of our findings and some implications for the future of research in the area of EI and RPV.

EI and RPV Analysis

The challenges surrounding EI are well-documented in the social science literature. Robinson (1950) pointed out that relying on aggregate data to infer the behavior of individuals can result in the ecological fallacy. Since then scholars have applied different methods to discern more accurately micro-level relationships from aggregate data. Goodman (1953, 1959) introduced ecological regression, where individual patterns can be drawn from ecological data under certain conditions. However, Goodman's statistical approach assumed that group patterns are consistent across each ecological unit and in reality that may not be the case.

Eventually, systematic analysis revealed that early methods could produce unreliable results (see, e.g., King 1997).[3] EI is King's (1997) solution to the ecological fallacy problem inherent in aggregate data.[4] Since the late 1990s, EI has been the benchmark method courts rely upon to evaluate RPV patterns in voting rights lawsuits. Indeed, according to the American Constitution Society for Law and Policy, EI is one of the three statistical analyses that must be performed in voting rights research on racial voting patterns (http://www.acslaw.org/sites/default/files/VRIGuidetoSection2Litigation.pdf).

Some critics, however, have asserted that King's model was designed primarily for binary data (2×2) such as situations in which just two groups (e.g., Blacks and Anglos; Hispanics and Anglos) exist. While many geographic areas (e.g., Mississippi, Alabama) still contain essentially two groups, the growth of ethnic/racial groups such as Latinos and Asians has challenged the historical biracial focus on race in the United States (Passel, Cohn, and Lopez 2011). To account for such complexities, Rosen et al. (2001) developed a hierarchical R×C approach, which they claim can be used to analyze multiple racial groups and multiple candidates together. However, due to the computationally intensive nature of their model, this approach was not initially employed that often in the social sciences, in general, and in voting rights cases in specific. In addition to this, King suggested that his method can still be used with more complex data (e.g., 3×2) by "iteratively" applying the model to different subsets of the data. In trying to assess voting patterns for three racial groups (Anglos, blacks, and Hispanics), the iterative technique would estimate three separate equations.

While this iterative technique has been widely used in voting rights cases, some social scientists have expressed concern. Ferree (2004), for instance, has argued that combining blacks and Anglos into a single "non-Hispanic" category in order to estimate Hispanic turnout may overestimate Hispanic turnout due to issues of aggregation bias and multimodality in the data. This suggests that the iterative approach could increase the likelihood of detecting RVP due to a larger-than-reality share of Hispanics in the data. While Ferree suggested some quick "fixes"—such as accounting for the relative size of each group or changing the order in which cells are estimated—to reduce aggregation bias and multimodality caused by collapsing rows or columns, she recommended estimating the cells of the R×C simultaneously rather than iteratively.[5] Others, such as Herron and Shotts (2003a, 2003b), have criticized EI estimates when used for second-stage regression, given that the error is incorporated into the second-level regression estimation.[6] Some have gone even further in arguing King's iterative approach can be "problematic and no valid statistical inferences can be drawn" and that only the hierarchical R×C approach developed by Rosen et al. (2001) can produce reliable estimates in multiethnic and multicandidate settings (Katz 2014).[7] In explaining the reasons of why the iterative EI technique is “ill-equipped” to handle complex data sets, Katz stated that “ . . . adding additional groups and vote choices to King’s (1997, p. 5) EI is not straightforward” and that “ . . . given the estimation uncertainty, it may not be possible to infer which candidate is preferred by members of the group” (p. 5). The argument against King’s iterative EI in the case of multiple racial group, or especially multiple candidate elections, is that EI pits candidate A versus all others who are not candidate A. If the election features four candidates (A, B, C, and D), some critics suggest that EI cannot accurately estimate vote choice quantities because vote for candidate A is compared against the combined vote for B, C, and D. Since the iterative approach would have to run four separate equations to obtain vote estimates for each candidate, social scientists such as Katz (2014) have even claimed in court that EI biases the findings in favor of bloc-voting: “ . . . this jerry rigged approach to dealing with more than two vote choices stacks the deck in favor of finding statistical evidence for racially polarized voting.”

Due to these concerns, advancements in computing power, and the availability of numerous packages developed for R, the computationally intensive R×C approach is now being used by some in place of the iterative EI. However, no study has empirically examined how these approaches perform side by side with real election data containing a number of different candidate and racial group combinations. Previous work has mostly leveraged Monte Carlo simulation or only a few election data sets (de Benedictis-Kessner 2015). Since we lack more expansive efforts to compare the two approaches, there simply is not enough information to enable researchers and legal practitioners to evaluate under which conditions the R×C method might perform differently than the iterative EI technique. For example, if there are three racial groups in equal thirds of the electorate, does aggregation bias create more error in the iterative EI than a scenario in which two dominant groups comprise 90 percent and a small group just 10 percent of the electorate? Likewise, is EI’s iterative approach to candidates more stable when analyzing three candidates and less stable when eight candidates contest the election? Is it really the case that the iterative approach is more likely than the R×C method to produce clear findings in favor of RPV patterns? The analytical task of this article is to consider these questions empirically; to systematically assess whether using the iterative EI method, as opposed to the hierarchical R×C method, can change the *substantive* conclusions one draws as it pertains to RPV patterns. Since we take advantage of real-world election data sets of varying electoral units and sizes, candidates, and racial/

ethnic groups that the courts would consider, our study provides a comprehensive attempt to answer some of the preceding questions.

Data and Methods

To examine how the two different methods process the same data sets, we rely on precinct voting data from three diverse states—California, Texas, and Florida—across 14 different elections from 2004 to 2012, in which a total of 78 candidates were on the ballot resulting in 193 race-candidate preference outcomes. For each of the 14 elections we analyze, we have precinct-level data on candidate vote distribution, as well as the racial demographics of the voting population in each precinct, and the total number of ballots cast. In two states, California and Florida,[8] we have data on voters by race and ethnicity. In Texas,[9] we have the number of eligible voters by race and ethnicity. Thus, the key variables are percent (candidate) and percent racial/ethnic group, and our estimates control for the number of total voters per precinct, as instructed by King (1997), Ferree (2004), and Rosen and colleagues (2001).[10]

The data we examine are diverse across almost any dimension as is illustrated by Table 1. We have data that range from more than 4,900 precincts in LA County to only 38 precincts in one school board district in Central Florida. The elections we examine also have varying number of candidates: from a head-to-head matchup with only 2 candidates to elections with up to 12 candidates. The data are also diverse with respect to the number of racial or ethnic groups within the electorate, starting with jurisdictions that are primarily Latino-Anglo, then areas with sizable Latino, Anglo, and Asian voting populations, and other geographies with Latino, Anglo, Asian, and Black voting populations. Thus, the data we consider are comprehensive and diverse across almost any metric, enabling us to follow a pattern of increasing complexity.

We begin the analysis with a basic data set with just 2 candidates and just two racial groups and then stick with these two racial groups and add election contests with 3, 4, 5, 6, 7, 9, and 12 candidates. In each election we analyze, there is at least one co-ethnic candidate, which allows us to assess RPV patterns. After comparing iterative EI and R×C results with two racial groups and multiple candidates, we turn to the analysis of multiple racial groups. We first assess only two candidates, but in two different environments with Latino, Anglo, and Asian and then Latino, Anglo, and Black. We then look at both multiethnic scenarios and contests with more than two candidates. Finally, we assess a very diverse electoral environment to further

Table 1. Summary Table of Elections Analyzed.

Geography	Year	Ethnic Groups	# Cand.	Contest	Precincts
Los Angeles County, CA	2010	2 (L, W)	2	Insurance Commissioner Dem Primary	4,980
Orange County, FL	2006	2 (L, W)	3	School Board	44
Corona, CA	2006	2 (L, W)	4	City Council	47
Orange County, FL	2012	2 (L, W)	5	County Commission	38
Corona, CA	2004	2 (L, W)	6	City Council	48
Fullerton, CA	2006	2 (L, W)	7	City Council	93
Vista, CA	2012	2 (L, W)	9	City Council	36
San Mateo, CA	2010	2 (L, W)	12	Superintendent of Public Education	433
Orange County, CA	2010	3 (L, W, A)	2	Insurance Commissioner Dem Primary	1,941
Fullerton, CA	2012	3 (L, W, A)	12	City Council	84
Harris County, TX	2010	3 (L, W, B)	2	Land Commissioner	885
Harris County, TX	2010	3 (L, W, B)	3	Lieutenant Governor Dem Primary	885
Orange County, FL	2008	3 (L, W, B)	4	Soil and Water Board of Directors	252
Los Angeles County, CA	2010	A (L, W, B, A)	7	Attorney General Dem Primary	4,974

Note: L = Latino; W = White; B = Black; A = Asian.

put the two methods to the test. We conclude with an analysis of a Democratic primary in LA County that featured seven candidates including viable Latino, Anglo, Black, and Asian candidates and provide results for all four racial groups of voters.

Before we proceed to the election data results, it is important to briefly underscore an important issue that researchers face when dealing with aggregate-level data, given that there are no bulletproof solutions to the problem of EI. Specifically, difficulties with calculating correct standard errors can arise if the aggregate data are not "informative" concerning the underlying microlevel data as detailed by Tam Cho and Gaines (2004). We emphasize this particular point to not only highlight the potential pitfalls of EI under certain conditions, which social scientists and legal practitioners should be aware of, but to also make the case that both iterative EI and R×C face similar constraints. That is, if a data set is "uninformative," both approaches will suffer and produce unreliable standard errors. Conversely, if a data set is amenable to EI (i.e., meets various model assumptions), both approaches will produce relatively accurate standard errors. Therefore, under both scenarios, a side-by-side comparison of the two approaches should result in drawing similar conclusions.

To gauge the level of information contained in a data set, it is recommended to examine tomography plots.[11] There are two specific diagnostic uses for tomography plots. By plotting all the logically possible pairs of parameter values—that is, the known information—tomography lines can succinctly display how constrained or flexible the parameters are and, thus, how difficult or easy the estimation problem will be. In a given plot, there is one tomography line bound with the [0,1] interval for each observation. Lines that do not extend across the entire unit square are further bounded than those that cross the entire unit square. If the lines are more bounded, one may be more successful when estimating the true parameter values (Tam Cho and Gaines 2004).

In addition to showing all the available deterministic information in a problem, tomography plots help assess whether the underlying truncated bivariate normal (TBVN) distribution imposed by EI is reasonable. If most of the tomography lines seem to intersect in a region, one may conclude that the actual individual-level data are most likely, but not certainly, clustered there, marking a potential location for the mode of the joint distribution of βs. However, if no area of intersection is evident and the parameter bounds are too wide, the implication is that the TBVN distributional assumption may not be entirely met. Stated differently, if the tomography plot is considered "uninformative," the data are less likely to have been generated from a TBVN distribution. This results in standard errors that may be too large to be useful or simply incorrect since they are computed based on the distributional assumption of the model (King 1997).

When using a tomography plot, it is important to keep in mind that the information obtained from this diagnostic plot is only suggestive. A tomography plot does not allow a researcher to make definitive claims about the particular distributional assumptions of the data. As Tam Cho and Gaines (2004) have stated, "... deciding whether a tomography plot is informative is something of an art, no one has devised a concrete measure for *'informativeness'* or any formal test for accepting or rejecting the TBVN distributional assumption (or any other distributional assumption) on the basis of the plot" (p. 155).

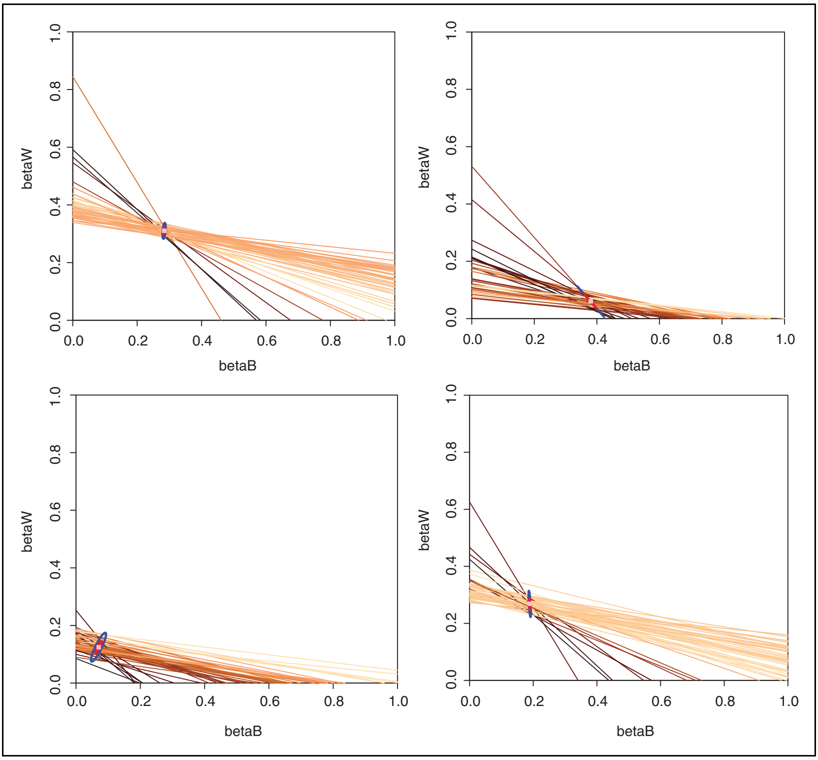

Figure 1. More "informative" tomography plots.

What this means is that tomography plots only provide an indication of the risk associated with forcing a distributional assumption on the data. If the parameter bounds are too wide and there is no general area of intersection, incorrect standard errors may be obtained (King 1997).

Despite the challenges that one faces when analyzing tomography plots, especially as the number of parameters increase, such inspection is worthwhile in helping researchers evaluate the extent to which the ultimate conditional distributions are fairly close approximations to the truth. If tomography plots lead one to reject the TBVN distributional assumption, the EI method may still be appropriate if one conditions on suitable covariates (Tam Cho and Gaines 2004).[12]

Our assessment of tomography plots suggests that some data sets are certainly more "informative" than others. For example, Figure 1

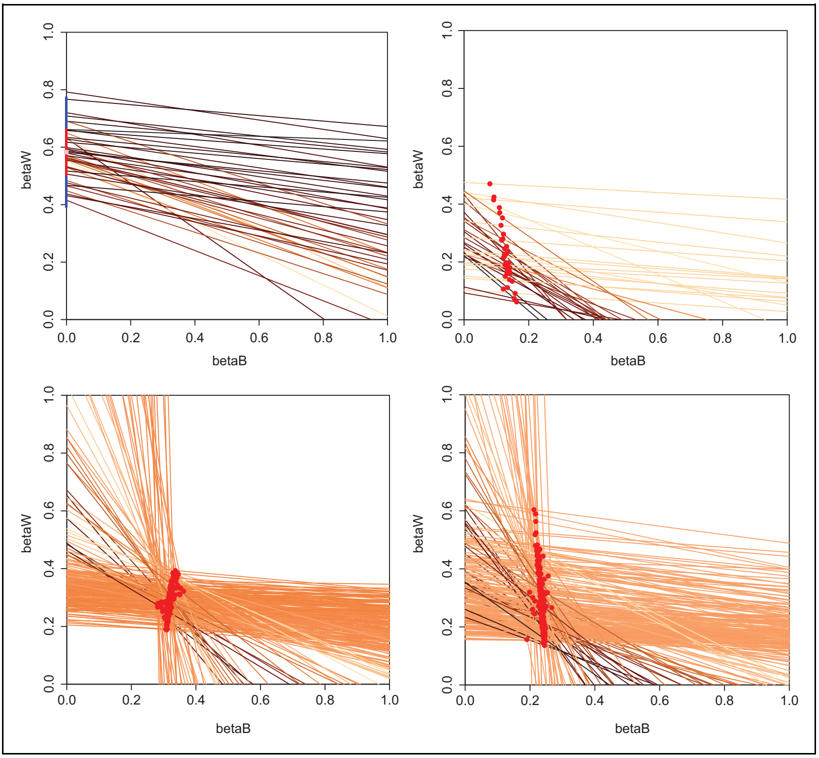

Figure 2. Less "informative" tomography plots.

demonstrates examples where the tomography lines tend to intersect in one general area, and the parameter bounds are fairly narrow in that they do not extend across the entire unit square of the plot. Based on these plots, one can make a reasonable case that the data have been generated from a TBVN distribution. In contrast, Figure 2 displays tomography plots that are considered less informative because the lines intersect in multiple areas or/and the parameter bounds are fairly wide. In cases in which the tomography plot indicates other distributional assumptions, the standard errors that one obtains may be less inaccurate.

The discussion surrounding the distributional assumptions imposed by EI leads to our key point: If the data sets are not consistent with the specified TBVN distribution, neither iterative EI nor EI R×C (or even Goodman's regression) will produce accurate standard errors unless one introduces relevant covariates into the model (Tam Cho and Gaines 2004). Thus, one cannot, on the basis of such diagnostics, make the claim that the R×C approach, which faces similar constraints as the iterative approach, somehow produces more or less accurate estimates. As the forthcoming results will demonstrate, a comparison of the two approaches yields similar substantive conclusions about the presence or absence of RPV regardless of the varying degrees of estimation difficulty.

Election Data Results

Using the R package eiCompare (Collingwood et al. 2016),[13] we estimated vote choice for candidates across racial groups using precinct-level election data. For EI, we took the iterative approach that has been questioned by some. In this approach, we iteratively estimated how each racial group voted for each candidate. That is, in an election with three different racial groups and seven different candidates, we estimated a total of 21 EI models. In contrast, the R×C approach allows analysts to estimate all the models simultaneously, although this method is very computationally intensive. Recall, our overarching question is: Does the iterative EI method substantively overestimate RPV compared to R×C?

Despite various claims regarding the potential limitations of the iterative approach, our analysis reveals that both methods lead analysts to similar conclusions about RPV across the 14 elections and 78 candidates we examined in the EI versus R×C approach. All the results race by race and candidate by candidate are reported in Online Appendix Tables 11–24 (which can be found at http://smr.sagepub.com/supplemental/). Where differences do emerge, they are often within only a few percentage points. In the 193 estimated racial group-candidate vote outcomes, we found that in 73 percent of the cases, the difference between EI and R×C was smaller than two points. More specifically, in 101 instances, the difference in the vote choice estimate was less than one point, and in 40 instances, the difference was between one and two points. This suggests remarkable consistency across the two approaches as it pertains to RPV analysis. For the remaining 27 percent of the cases, only 11 of them—or 6 percent—produced estimates that were over six points different from one another, as summarized in Table 2. Even in these 11 instances, the two models resulted in the same conclusions of preferred candidates and the presence or absence of RPV.

We also did not find any convincing evidence that EI will lead analysts to reach conclusions in favor of RPV. For example, in the first election (2 × 2) we considered, EI reports almost identical minority cohesion—84.89 (EI) to

Table 2. Distribution of Difference between EI and R×C Vote Choice Estimates.

EI versus R×C Outcomes	*N*	Percentage
Less than one point difference	101	52.3
One to two points difference	40	20.7
Two to four points difference	28	14.5
Four to six points difference	13	6.7
Over six points difference	11	5.7

Note: Of 193 vote choice scenarios. EI = ecological inference; R×C = rows by columns.

Table 3. Comparison of Racially Polarized Voting Results across EI and R×C.

	EI and R×C Identified	EI and R×C Identified	EI and R×C Identified	EI and R×C Identified
Voting Bloc	Same #1 Ranked Candidate	Different #1 Ranked Candidate	Same #2 Ranked Candidate	Different #2 Ranked Candidate
Minority voters	21	0	20	1
White voters	15	0	14	1

Note: EI = ecological inference; R×C = row by column.

84.50 (R×C)—for the Latino-preferred candidate. This is consistent with the overall patterns we previously reported. The vast majority of the estimates fall within one or two points of one another. However, there are some cases in which EI produces slightly higher minority vote cohesion estimates, typically around three to five percentage points higher. Yet the differences are substantively not meaningful because both methods clearly identify the same preferred or first choice candidate. Table 3 illustrates that as far as RVP determinations are concerned, there is not a single instance across all election types with varying degrees of precincts, candidates, and racial groups that EI and R×C point analysts to different first choice candidates. Both methods are also highly consistent in identifying second-choice candidates. When observing minority vote cohesion estimates, both EI and R×C point to the same second-choice candidate in 20 of 21 cases. Likewise, we found no evidence that Anglo bloc voting against minority-preferred candidates is stronger under EI as compared to R×C. Here again, both methods identify the same

Table 4. Elections with Two Groups (Latino and Non-Latino).

		EI versus R×C Difference	
Geography	# of Candidates	Latino	Non-Latino
Los Angeles County, CA	2	−0.39	−0.36
Orange County, FL	3	−10.69	−1.18
Corona, CA	4	−2.21	0.56
Orange County, FL	5	−0.97	−0.36
Corona, CA	6	−0.09	1.14
Fullerton, CA	7	−3.84	0.19
Vista, CA	9	−5.36	0.81
San Mateo, CA	12	−6.36	−0.14

Note: Differences for each group's preferred candidate. EI = ecological inference; R×C = row by column.

first choice candidates for Anglo voters in all cases and only disagree in one instance on the second-choice candidate. Overall then, even where differences emerged, they were often negligible and would round to the same whole number or substantively not meaningful for RPV determinations. That is, scholars or judges evaluating the results would not contend that the two methods produced different vote preference rankings.

Recall that our second research question was: Are there systematic outcome differences between EI and R×C when analyzing elections with few candidates versus elections with multiple candidates? We might expect greater differences to emerge when there are more candidates than fewer candidates—the claim is that R×C is designed for this scenario whereas EI is more equipped in dealing with 2 × 2 data sets. Another way of stating this is: Do EI and R×C essentially produce substantively similar results when there are 2, or maybe 3 candidates, but start to diverge when 6, 7, or more than 10 candidates are on the ballot?

In the first section of our analysis, we compared EI and R×C with only two racial groups—Latinos and non-Latinos—across eight elections in which the number of candidates on the ballot varied from 2 to 12. The elections consisted of contests in Los Angeles, CA; Orange County, CA; Corona, CA; Orange County, FL; Corona, CA; Fullerton, CA; Vista, CA; and San Mateo, CA. This diversity allowed us to assess whether the number of candidates impacted the stability of EI and R×C estimates. Table 4 reports the difference between the two methods in vote choice for each group's preferred candidate across all eight elections analyzed. Figure 3 visualizes

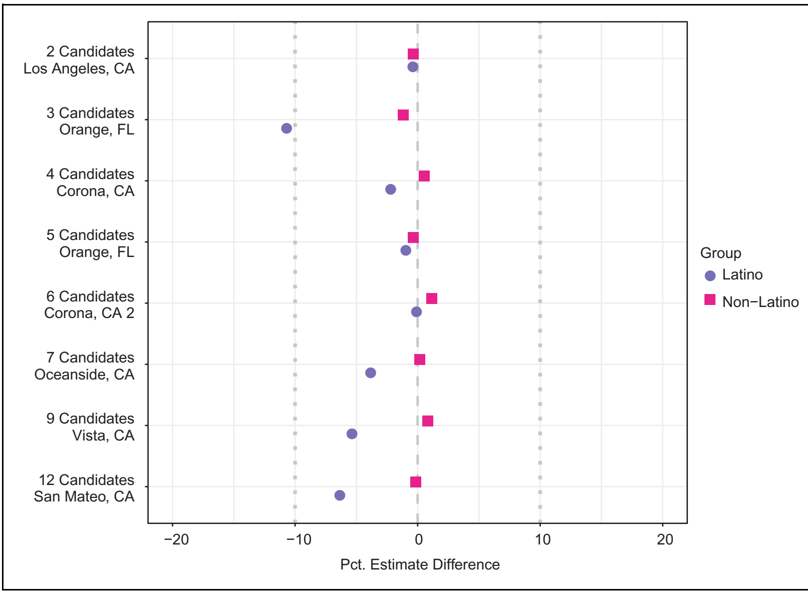

Figure 3. Ecological inference versus row by column differences, elections with two groups (Latino and non-Latino).

the differences between method estimates by group for each election. As is illustrated, there is no detectable pattern that would lead one to conclude that the iterative EI is more likely to produce results in favor of RPV. Furthermore, even when the data sets were more or less amenable to EI based on an assessment of tomography plots, the conclusions regarding RPV did not change. For instance, in the Vista, CA, election results, the data set was considered more "informative" in that parameter bounds were relatively narrow and a general area of intersection existed. In contrast, the LA and San Mateo elections were cases in which the data sets were considered less informative. Nevertheless, both approaches produced similar outcomes. That is, no patterns were detected with more or less informative data sets, given that both methods face similar estimation constraints if certain conditions, such as the TBVN distributional assumption, are not met.

So far we have only examined races with two groups (Latino and non-Latinos/Anglos). In the next section, we compare EI and R×C in six elections with more than two racial groups; two elections with Latinos, Asians, and Anglos; three with Latinos, Blacks, and Anglos; and one election with

Table 5. Elections with Three Groups (Latino, Black, and White).

		EI versus R×C Difference		
Geography	# of Candidates	Latino	White	Black
Harris County, TX	2	6.38	0.78	1.87
Harris County, TX	3	0.61	0.83	4.47
Orange County, FL	4	0.19	−1.21	1.62

Note: Differences for each group's preferred candidate. EI = ecological inference; R×C = row by column.

Table 6. Elections with Three Groups (Latino, Asian, and White).

		EI versus R×C Difference		
Geography	# of Candidates	Latino	White	Asian
Orange County, CA	2	1.83	0.50	6.40
Fullerton, CA	12	−4.06	0.48	−2.84

Note: Differences for each group's preferred candidate. EI = ecological inference; R×C = row by column.

Table 7. Elections with Four Groups (Latino, Black, Asian, and White).

		EI versus R×C Difference			
Geography	# of Candidates	Latino	White	Asian	Black
Los Angeles County, CA	7	0.005	−0.07	−4.46	4.22

Note: Differences for each group's preferred candidate. EI = ecological inference; R×C = row by column.

the four racial groups. This allows us to assess our third major question: Are there systematic outcome differences between EI and R×C when analyzing elections with more than two racial groups?

In addition to examining elections with different racial group combinations, our data enabled us to consider elections with as low as 2 and as high as 12 candidates so that we can continue to assess whether systematic differences emerge between EI and R×C in much more complex environments. Tables 5–7 report the estimate vote difference between the two methods by each group's preferred candidate for each one of the elections examined.

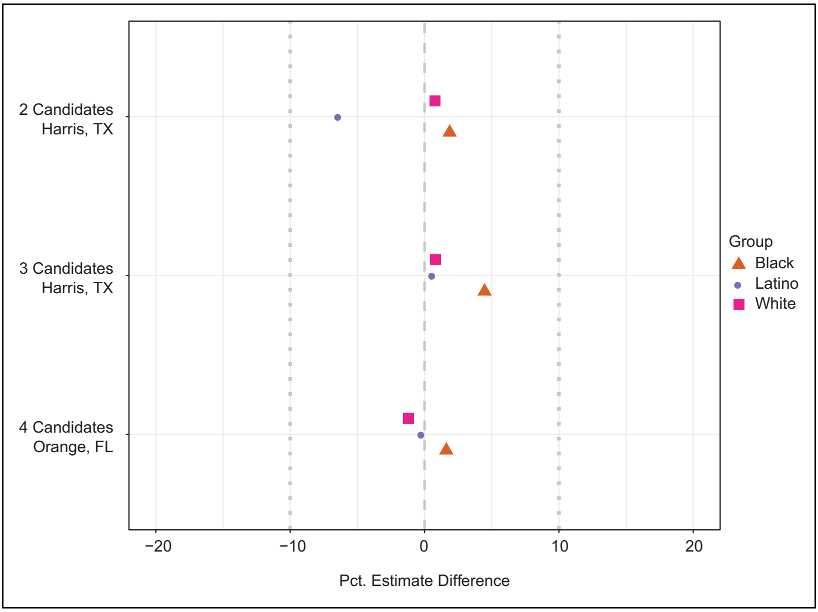

Figure 4. Ecological inference versus row by column differences, elections with three groups (Latino, white, and black).

Similarly, Figures 4 and 5 visualize the differences. Finally, Figure 6 presents a compiled visualization of all the races with more than two ethnic groups. The results display remarkable similarity between EI and R×C estimates even as the number of ethnic groups and candidates increase. Once again, we did not detect any patterns that would lead analysts to conclude that EI is more or less likely than R×C to produce results in favor of or against RPV.

Comparison with Exit Poll Data

In many, if not most, situations where analysts are called to evaluate the presence or absence of RPV, EI is the chosen method in part because individual-level polling data are unavailable. For instance, pollsters do not collect data for elections in small cities, such as Corona, CA. In large cities, though, exit poll data are occasionally available.

While our main question is whether EI and R×C produce substantively similar RPV outcomes, there is a possibility that EI may be inaccurate relative to the "truth" more often than the R×C approach. To consider this

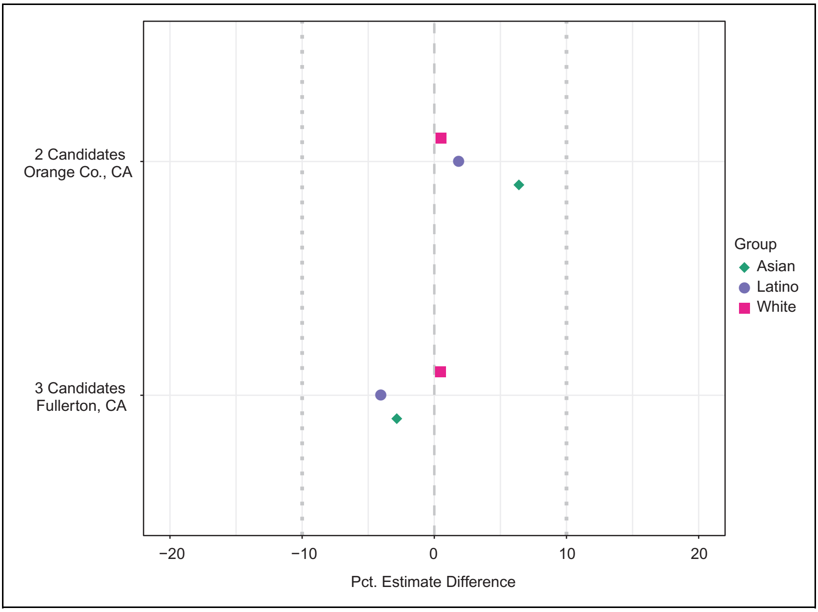

Figure 5. Ecological inference versus row by column differences, elections with three groups (Latino, white, and Asian).

possibility, we compare EI and R×C estimates in a voting scenario with known outcomes that provide vote choice by race/ethnicity (i.e., an exit poll or preelection poll). To be sure, exit polls can produce biased estimates of subgroups because of the reliance on "bellwether" counties or precincts comprising heterogeneous populations including racial/ethnic groups (Barreto et al. 2006; Mitofsky 1998; Traugott and Price 1992). Specifically, Barreto et al. (2006) argue that heterogeneous precinct-based exit polls often overestimate conservative voting among Latino voters because pollsters selecting bellwether precincts are more likely to encounter acculturated Latinos who are disproportionately Republican. That said, an exit poll is still another point of comparison employed to get closer to the actual individual-level voting behavior.

Many studies have pointed out that ecological fallacy and other estimation issues can produce EI results that are unreliable. While we acknowledge the limitations of EI, we find that the results from EI and R×C are similar to the individual-level exit poll data as it pertains to evaluating RPV patterns in

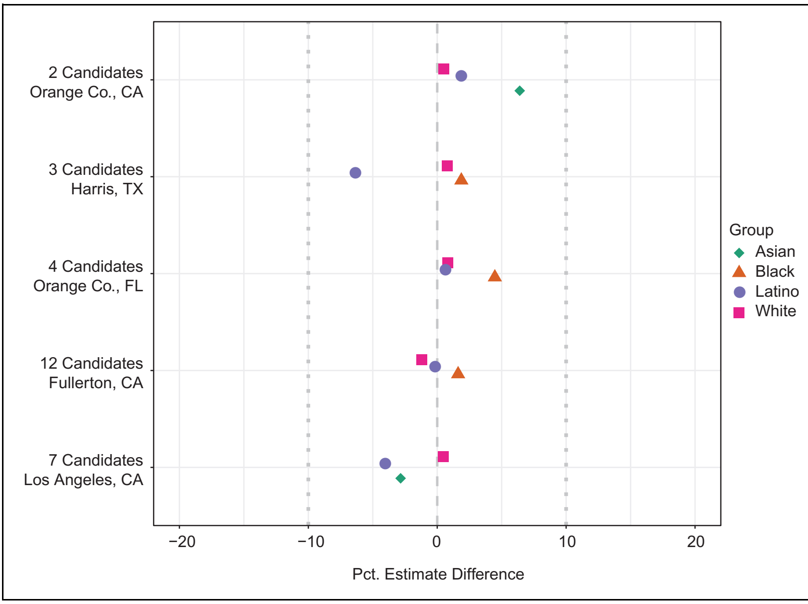

Figure 6. Ecological inference versus row by column differences, elections with four groups (Latino, White, Black, and Asian).

VRA cases. Table 8 presents EI and R×C estimates for the 2005 LA mayoral runoff election between Antonio Villaraigosa (Latino) and James Hahn (Anglo). These estimates are compared to results from the LA Times exit poll. Our findings demonstrate that not only do EI and R×C produce remarkably similar estimates, but that the results closely match the individual-level estimates from the LA Times poll. More specifically, the EI method estimates Villaraigosa receiving 83 percent of the Latino vote and only 44 percent of the Anglo vote; the R×C method estimates Villaraigosa receiving 82 percent of the Latino vote and just 48 percent of the Anglo vote. If the task is to evaluate a pattern of RPV, both methods closely match the conclusion one would draw from the exit poll, which reports that an estimated 84 percent of Latinos voted for Villaraigosa while only 50 percent of Anglo voters made the same choice. Moreover, the EI and R×C estimates are all within the margin of error of the individual-level data reported by the LA Times exit poll. In sum, this comparison provides additional evidence that both methods may be useful in evaluating RPV in VRA cases.

Table 8. Percent Voting for Antonio Villaraigosa (AV) and James Hahn (JH) by Racial Group.

Race	EI: AV	EI: JH	R×C: AV	R×C: JH	Exit: AV	Exit: JH	MOE
White	44	56	48	52	50	50	±2.5
Black	57	43	51	49	48	52	±4.2
Latino	83	17	82	18	84	16	±3.6
Asian	48	52	47	53	44	56	±6.1

Note: Comparison between EI, R×C, and exit poll results, Los Angeles Mayoral Election Runoff, May 2005. Exit poll data from Los Angeles Times. EI = ecological inference; MOE = margin of error; R×C = row by column.

Monte Carlo Simulation Results

While the analyses with real-world election data demonstrated congruence between the two methods, Monte Carlo simulations provide another way of evaluating our most basic question: Do analysts reach substantively different conclusions when comparing iterative EI and R×C estimates?[14] To answer this question, we drew simulations from a β distribution with parameters $\beta = 2$, $\alpha = 2$ to construct the following data sets: two candidates, two groups; two candidates, three groups; three candidates, two groups, three candidates, three groups; and four candidates and four groups. Each data set contains anywhere from 100 to 1,000 precincts, and each precinct ranges in size from 10 to 1,000 total voters. The data also contain a set of columns for each group's simulated percent share of the precinct and percent vote for the hypothetical candidates. For each of the data set types (2×2, 2×3, 3×2, 3×3, and 4×4), we then randomly generated 100 data sets, estimated group votes using both iterative EI and R×C methods, and stored the average difference between the two methods across all groups and candidates. Figure 7 visually depicts the simulation results.

The findings largely validate the results obtained with real-word election data. Across 500 randomly generated data sets, we find tremendous consistency between the two methods, with overall mean differences by each election type ranging between one and four percentage points. In voting rights cases, these observed differences would almost never alter one's substantive conclusions about RPV patterns. Even in the rare cases where we found larger discrepancies (e.g., only 9 percent of the 2×3 data types), both methods concurred on the hypothetical groups' preferred candidate. A detailed look at the results, for instance, revealed that iterative EI estimated that 80 percent of group 1 favored candidate 2, while R×C estimated that 90

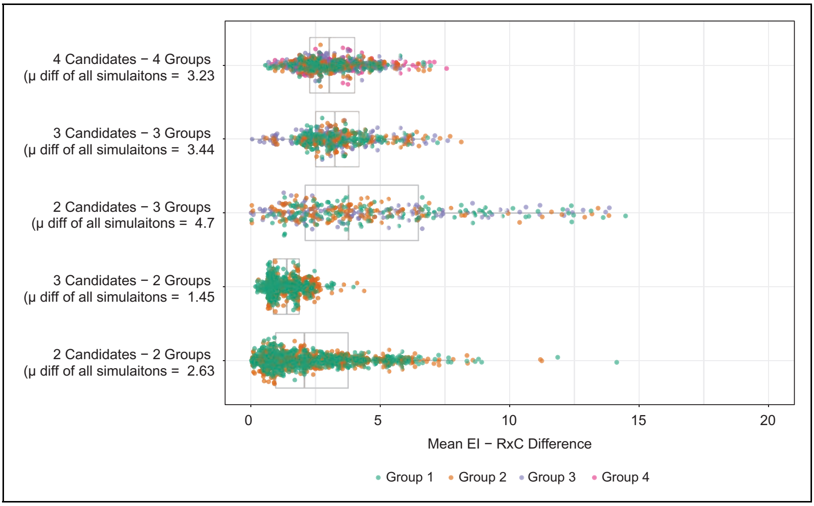

Figure 7. Simulated data results (500 data sets).

percent of group 1 favored the same candidate. Thus, for all practical purposes, experts would reach similar conclusions about RPV as the two methods concurred on the direction and degree of polarization.

Finally, and perhaps most importantly, our assessment of the simulation results did not reveal any systematic patterns where iterative EI produced higher or lower estimates than the R×C method. In some cases, R×C produced higher group estimates while in other cases iterative EI did.

Model Congruence Score (MCS): Do the Two Methods Lead to Similar Substantive Conclusions?

The previous sections demonstrated that iterative EI and R×C tend to produce substantively similar vote choice estimates under various conditions. However, our discussion of the "substantive" evaluation of the results did not provide a systematic way of interpreting the findings. A systematic evaluation of congruence between the two methods is important because plaintiffs must show courts that RPV exists, and that RPV is not just a function of choosing one statistical method over another, but something that generally holds regardless of the approach. Social scientists are also similarly interested in understanding the extent to which results are substantively consistent across different estimators.

To this end, we introduce a new approach to aid analysts in determining whether the two methods produce similar judgments, which we call the MCS. The MCS can be applied in either 2×2 settings or with some adjustments extended to situations with multiple candidates and multiple groups, although analysts may want to set some decision rules in terms of whether to combine all candidates of the same race together (e.g., one election might have multiple white/Anglo candidates: Smith, Toms, and Johnson) into one racial group candidate for the purposes of assessing RPV patterns.[15]

What exactly can the MCS reveal with respect to voting right analysis? First, do both iterative EI and R×C conclude that minority voters prefer the minority candidate and that Anglo voters prefer the minority candidate? If minority voters prefer the Anglo candidate and so do Anglo voters, then RPV does not exist. Likewise, if both minority and Anglo voters both prefer the minority candidate, RPV does not exist. Both cases would not meet the Gingles threshold outlined by the court. To answer this initial question, the MCS rates whether simple polarized voting exists based on the estimates obtained from iterative EI and R×C.

Second, what is the relative degree of RPV in each of the models? For example, do both models suggest a 30-point gap in racial voting preference, or does one model suggest only a 5-point difference and the second model suggests a 40-point difference? The difference in voting preferences, and not just the direction of preferences, is a very important component of the congruence score and informative to the courts. In order to answer this second question, MCS first estimates the percentage point gap between minority and Anglo voters for the minority preferred candidate and then for the Anglo-preferred candidate. Next, MCS evaluates what percentage of minority voters would need to switch from voting for the minority candidate to supporting the Anglo candidate such that there is an even 50–50 distribution, and no clear preferred candidate. Likewise, MCS calculates the percentage of Anglo voters that would need to switch from voting for the Anglo candidate to supporting the minority candidate to create a 50–50 distribution. While the formula is different, the logic behind this measure is similar to the dissimilarity index commonly used in demography (Massey and Denton 1988).

Third, if voting patterns hold, are minority voters blocked by Anglo voters from electing a minority candidate? And by how much are they blocked? Again this step adds both a simple "yes/no" distinction of being blocked but also calculates and compares the degree by which a minority-preferred candidate is blocked. Overall, the MCS attempts to provide a simple measure, ranging from 0 to 1, to assess how much congruence exists between and within the vote choice estimates across iterative EI and R×C.

Table 9. 2 × 2 Congruence Results for Los Angeles County Insurance Commissioner Election 2010.

Race	EI	R×C	Congruence
MV1-WV for MC1	63.2	62.4	0.988
MC1 preferred by MV1	Yes	Yes	1
MC1 preference rate	34.9	34.5	0.988
MC1 blocked by WV	Yes	Yes	1
MC1 block rate	−28.3	−27.9	0.987
MC model congruence			0.993
MV1-WV for WC1	−63.2	−62.4	0.987
WC1 preferred by WV1	Yes	Yes	1
WC1 preference rate	28.3	27.9	0.987
WC1 blocked by MV1	Yes	Yes	1
WC1 block rate	−34.9	−34.5	0.988
WC model congruence			0.992
Total model congruence score			0.992

Note: Congruence calculations based on Online Appendix Table 11 (which can be found at http://smr.sagepub.com/supplemental/) findings. EI = ecological inference; R×C = rows by columns.

We first calculated MCS for both iterative EI and R×C in a simple 2 × 2 configuration to show in more detail how the process works. We report congruence scores for each metric, which is scaled from 0 to 1, where 0 reveals the two methods are in complete disagreement and 1 indicates the two methods are in complete agreement. For ease of interpretation, we explain the precise metrics for the aforementioned three tests and their congruence with actual data from iterative EI and R×C estimates of the Latino and non-Latino vote from the 2010 LA County Insurance Commissioner race where the Latino candidate, De la Torre, ran against Jones (Anglo). While the non-Latino group includes non-Latino minorities, for simplicity, we bin Anglos with non-Latino minorities in order to craft a simple 2 × 2 scenario (see Table 11 in the Online Appendix [which can be found at http://smr.sagepub.com/supplemental/] for full vote choice estimates).

To assess whether Latino voters prefer the Latino candidate, we examine the difference between Latino support for De la Torre and Anglo/other support for De la Torre. According to Online Appendix Table 11 (which can be found at http://smr.sagepub.com/supplemental/), iterative EI places Latino support for De la Torre at 84.9 percent, whereas for Anglo/non-Latinos, the estimate is at just 21.7 percent. The gap in candidate support by racial group is thus just over 63 percent, which is shown in column 2, labeled EI, the first row of Table 9. The same calculation is made for the

R×C method, placing Latino support for De la Torre at 84.5 percent and the non-Latino support at 22.1 percent—a difference of 62.4 percentage points. How similar are these findings? To calculate the congruence score on this measure, we take the absolute difference between the iterative EI and R×C estimate for Latino and non-Latino support for De la Torre, then divide this by the absolute mean difference of the two methods. Finally, to transform this into a 0–1 scale, where 1 equals complete congruence and 0 equals no congruence, we subtract the resulting value from 1 so that values closer to 1 imply higher congruence:

$$\begin{aligned} x &= \text{EI Latino vote for De la Torre} - \text{EI Non-Latino vote for De la Torre}, \\ \text{y} &= \text{R} \times \text{C Latino vote for De la Torre} - \text{R} \times \text{C Non-Latino vote for De la Torre}, \\ &= 1 \quad \frac{abs(x-y)}{abs(\text{mean}(x,y))}. \end{aligned} \tag{1}$$

We can plug the data from Online Appendix Table 11 (which can be found at http://smr.sagepub.com/supplemental/) into the equation above to produce the congruence score, which is identical to the congruence score appearing on row 1 of Table 9:

$$\begin{aligned} &= 1 \quad \frac{abs((84.89 - 21.74) - (85.50 - 22.12))}{abs(\text{mean}((84.89 - 21.74), (85.50 - 22.12)))}, \\ &= 0.988. \end{aligned} \tag{2}$$

Row 2 in Table 9 assesses whether De la Torre is preferred by Latino voters. The congruence receives 1 if both the iterative EI and R×C method reveal that Latinos preferred De la Torre to Jones (or 1 if both methods revealed a preference for Jones). In the present case, both methods show that Latinos prefer De la Torre, so the congruence on this metric receives a 1. The preference rate is calculated as the difference between Latino support for the Latino candidate, De la Torre, and the Anglo candidate, Jones. For iterative EI, this would be 84.89 − 15.05. The resulting figure is then divided by 2, to show how much above the 50 percent mark De la Torre is preferred over Jones. In other words, what is the percentage of Latino voters who would have to switch to Jones so that Latinos did not prefer either candidate? For iterative EI, this number is 34.9. Using the same calculation for R×C, we arrive at 34.5. Thus, our numbers in this case are very similar, and so a congruence score of 0.988 is reported. The equations for this congruence are listed below:

$$\begin{aligned} x &= (\text{EI Latino vote for De la Torre } - \text{ EI Latino vote for Jones})/2, \\ y &= (\text{R} \times \text{C Latino vote for De la Torre } - \text{R} \times \text{C Non-Latino vote for Jones})/2, \\ &= 1 \quad \frac{abs(x-y)}{abs(\text{mean}(x,\ y))}. \end{aligned} \tag{3}$$

The actual numbers are presented here:

$$\begin{aligned} x &= (84.89 \ - \ 15.05)/2, \\ y &= (884.50 \ - \ 15.50)/2, \\ &= 1 \quad \frac{abs(x-y)}{abs\Big(\text{mean}(x,\ y)\Big)}, \\ &= 0.988. \end{aligned} \tag{4}$$

Finally, we turn to vote blocking. Given the way districts are often drawn, this is a crucial question posed to judges who assess whether Anglos are blocking Latinos from electing their preferred candidates (usually Latino). In our working example, for non-Latinos, we subtract their support for Jones from non-Latino support for De la Torre. This is then divided by two (as in the above set of equations). This essentially measures how much Anglos (or non-Latinos) support the Anglo candidate and how many votes they would need to dole out to the Latino candidate to not block the Latino candidate from getting elected. For iterative EI, this is (21.7 − 78.2)/2, and for R×C, this is (22.1 77.9)/2. Once again, the congruence score is calculated in a similar way as above, which produces a score of 0.987. Row 4 of Table 9 also reports whether Anglos are, in general, block voting against Latinos—if both the iterative EI and R×C agree, then the congruence is given a 1.

$$\begin{aligned} x &= (\text{EI Non-Latino vote for De la Torre } - \text{ EI Non-Latino vote for Jones})/2, \\ y &= (\text{R} \times \text{C Non-Latino vote for De la Torre } - \text{ R} \times \text{C Non-Latino vote for Jones})/2, \\ &= 1 \quad \frac{abs(x-y)}{abs\Big(\text{mean}(x,\ y)\Big)}. \end{aligned} \tag{5}$$

$$\begin{aligned} x &= (21.74 \ - \ 78.24)/2, \\ y &= (22.12 \ - \ 77.88)/2, \\ &= 1 \quad \frac{abs(x-y)}{abs\Big(\text{mean}(x,\ y)\Big)}, \\ &= 0.987. \end{aligned} \tag{6}$$

Table 10. Summary of Overall Model Congruence Scores (All Elections Analyzed).

R×C	Geography	Precinct (*n*)	Congruence
2×2	Los Angeles, CA	4,980	.992
2×3	Orange County, FL	44	.925
2×4	Corona, CA	47	.840
2×5	Orange County, FL	38	.956
2×6	Corona, CA	48	.855
2×7	Fullerton, CA	93	.940
2×9	Vista, CA	36	.887
2×12	San Mateo, CA	433	.917
3×2	Orange County, CA	1,941	.882
3×12	Fullerton, CA	84	.857
3×2	Harris County, TX	885	.935
3×3	Harris County, TX	885	.917
3×4	Orange County, FL	252	.914
4×7	Los Angeles, CA	4,974	.868

For the total Latino candidate congruence score, we take the mean of the existing congruence scores, resulting in a final score of 0.993. The process is reversed for calculating the requisite scores for the Anglo candidate. In the 2 × 2 scenario, the numbers are essentially the same as those calculated for the minority candidate; however, the coefficient sign is switched, and the block rate and preference rates are swapped. The final step taken to obtain an "overall" or "total MCS" is to then calculate the average of the minority and Anglo candidate congruence scores obtained in the previous steps.

Beyond the 2 × 2 example, we also provide detailed MCSs in the Online Appendix (which can be found at http://smr.sagepub.com/supplemental/) for a 2 × 4, 2 × 5, 3 × 3, and 4 × 7 election analysis comparing iterative EI and R×C. For ease of interpretation, Table 10 summarizes the total congruence scores for all elections analyzed. Overall, the findings demonstrate high levels of congruence across a variety of different elections with multiple candidates and multiple racial/ethnic voter groups.

Conclusion

This article engages with an important methodological topic with real-world implications. Specifically, we examined three questions to assist social scientists, legal practitioners, and the courts working with VRA cases in which only aggregate-level (e.g., precinct) data exist: (1) Does EI's iterative technique substantively overestimate RPV compared to R×C? In other words, does EI lead analysts to detect RPV while R×C does not? (2) Are there systematic outcome differences between EI and R×C when analyzing elections with few candidates versus elections with multiple candidates? (3) Are there systematic outcome differences between iterative EI and R×C when analyzing elections with more than two racial groups? These questions were assessed with real-world data from 14 elections with 78 candidates and 193 race-candidate vote choice outcomes, and 500 simulated data sets of varying number of candidates and groups.

To examine whether voting districts experienced RPV, we estimated vote shares for different candidates from voters of different racial/ethnic groups using two of the most commonly used EI methods. We evaluated King's iterative EI approach against the more recent R×C approach. Using elections with multiple candidates and multiple groups (i.e., Latinos, Anglos, Blacks, Asians), we did not find significant differences between the two methods in terms of estimating candidate support. To the extent that differences did emerge, they were not systematic in any way and did not alter our substantive conclusions of the overall results. Furthermore, in one analysis where exit poll data were available, we compared the iterative EI and R×C results against known exit poll figures and found that the three methods produced statistically and substantively indistinguishable candidate estimates for different racial/ethnic voting blocs. A series of Monte Carlo simulations provided additional support for the assertion that iterative EI and R×C produce substantively similar estimates in different candidate–group combinations.

Finally, we presented a new congruence test that analysts can implement to interpret RPV patterns when using both iterative EI and R×C methods. We outlined how analysts can calculate MCS ranging from 0 to 1, where 0 indicates iterative EI and R×C produce completely opposite results and 1 indicates that the methods are in complete agreement. We then applied this test to a host of elections, finding that overall congruence between the two techniques is very high. In other words, an MCS analysis provides a quantitative figure to assess EI/R×C congruence; in the present scenario, these figures suggest no meaningful differences between the two methods.

Our findings have important implications for academics and practitioners who are involved in voting rights litigation. While there has been a robust debate on precisely what method to use, we suggest that claims about the superiority of one method over the other should not be made without clear and convincing evidence. While we find no concerning discrepancies between the two methods in the elections we analyzed, we do not claim that our analysis rests all debate. Rather, we invite social scientists to further examine the different approaches as they pertain to identifying the presence or lack of RPV patterns.

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Supplemental Material

Supplemental material for this article is available online.

Notes

1. To be clear, the principal aim of the present article is not to settle the debate on the accuracy of ecological inference (EI) in the sciences writ large (e.g., see Frair et al. 2010; Freedman 1999; Greenland 2001; Martin et al. 2005; Tam Cho and Gaines 2004; Wakefield 2004), but rather to assess the degree of similarity or difference with respect to two heavily used methodologies the courts rely upon to decide whether jurisdictions are systematically discriminating against minority voters.
2. The courts still do, however, rely on bivariate correlation, Goodman regression, and homogeneous precinct analysis. To this end, we have incorporated the Goodman regression into our R package so that analysts can assess this method alongside iterative EI and row by column (R×C).
3. However, in an extensive review, Owen and Grofman (1997) concluded that despite some valid theoretical concerns, the single-equation ecological regression still holds up and provides meaningful and accurate estimates as it pertains to RPV. A decade later, Grofman and Barreto (2009) evaluated how ecological models compare to one another using a combination of simulated data, actual election precinct data, and an accompanying exit poll. Their analysis demonstrated that there is general consistency across the single and double equation methods and that once voter turnout rates are accounted for similar conclusions are reached.

4. It should be noted that EI has faced some criticism, especially in the fields of biological sciences, ecology, epidemiology, and public health (Frair et al. 2010; Freedman 1999; Greenland 2001; Martin et al. 2005; Wakefield 2004). However, within the subfield of racial voting patterns in American elections, EI is still heavily relied upon, particularly by the courts.
5. The simultaneous method recommended is Rosen et al.'s (2001) R×C method.
6. In response to this issue, Adolph and King (2003) adjusted the EI procedure to reduce inconsistencies when estimating second-stage regressions.
7. Greiner and Quinn (2010) combined R×C with individual-level exit poll data and showed that a hybrid model is perhaps even more preferable than a straight aggregation model. However, using exit poll data is not always available to researchers and practitioners. Indeed, in most county or city elections, exit poll data do not exist, which is why scholars often attempt to infer voting patterns with aggregate data.
8. In California, we have individual-level race estimates based on surname analysis by the UC Berkeley Statewide database. In Florida, we have individual-level race from the voter registration application as a result of the Voting Rights Act.
9. In Texas, we have Census citizen voting age population (CVAP) data on the racial distribution by precinct voter turnout data (VTD) from the Texas Legislative Council.
10. All the election data and R code to reproduce the election results are available at https://www.collingwoodresearch.com/data.html.
11. Note here that as the number of parameters increase, tomography plots will become very difficult to analyze and, thus, lose their diagnostic value.
12. Therefore, tomography plots can also be viewed as a diagnostic tool for determining the necessity of adding appropriate covariates to the model. The tomography plots do not necessarily need to be included as Appendix materials, but analysts may consider evaluating them during the analysis stage.
13. At the time of this publication, we used eiCompare version 2.4, available at https://github.com/lorenc5/eiCompare.
14. However, we note that simulations are not necessarily a "better approach" since randomly generated data could contain many scenarios in which there are no clear minority-preferred candidates—that is, cases that are of little interest to potential plaintiffs.
15. We use the term white to mean Anglo in all tables and figures.

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